# Quantum Chemical Calculations of Topology of Molecular Systems. I. The Degenerate Rearrangement in Cyclopentadiene and Cyclopentadienylsilane 

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#### Abstract

A potential curve close to the reaction coordinate for the [1,5] rearrangement in cyclopentadiene and in cyclopentaaienylsilane is calculated. A fast-convergence optimization procedure is elaborated for the computation of a geometry of the points on the curve. Scanning the calculated points and processing these with the corresponding orbitals method allow us to distinguish three regions on the reaction coordinate: (A) minima, where energy is a quadratic function; (B) descents, where the energy change is practically linear; (C) passes, where energy is almost constant. At the A and B regions the total energy is to the greatest extent affected by the bending angles of the methylene hydrogens with respect to the ring plane. The "permissibilities" of the transition from the ground state to the regions A and B do not undergo any significant change. At the region C the "permissibilities," the ring $\pi$ orbitals, the $\mathrm{C}-\mathrm{C}$ bonds, and the migrating $\mathrm{H}-\mathrm{C}$ distance change strongly. The calculated activation energy is $10.2 \mathrm{kcal} / \mathrm{mol}$ with CNDO$/ 2$ and $18.4 \mathrm{kcal} / \mathrm{mol}$ with MINDO $/ 2$ methods; the experimental value is $24 \mathrm{kcal} / \mathrm{mol}$. There are significant differences between the potential surface energy patterns calculated via these two methods (e.g., the CNDO/2 overestimates the pass length); however, the optimized geometries are quantitatively identical. For a cyclopentadienylsilane molecule, the calculated energy of activation is $6 \mathrm{kcal} / \mathrm{mol}$, one-half the experimental energy. "Permissibility" of the rearrangement is 0.749 for $\mathrm{C}_{5} \mathrm{H}_{6}$ and 0.798 for $\mathrm{C}_{5} \mathrm{H}_{5} \mathrm{SiH}_{3}$. The main spatial factor which governs the rearrangement course is the alteration of bending angle formed by the $\mathrm{Si}-\mathrm{C}$ bond and the $\mathrm{C}_{5} \mathrm{H}_{5}$ ring plane, by reference to the methylene angle in $\mathrm{C}_{5} \mathrm{H}_{6}$. The calculation demonstrates that the channels of the $[1,1]$ and $[1,3]$ rearrangements, forbidden through symmetry and spatial factors cannot be related to any potential curve which would be close to the reaction coordinate.


Experimental investigations of a reaction mechanism allow one just to make indirect prediction of the geometry of a metastable intermediate or of the activated complex. Quantum chemical calculations may help to go beyond this and provide insights not yet accessible experimentally. In order to calculate a chemical reaction in some detail one should be able, in the first place, to calculate quantitatively the geometry of stable molecules. It has been shown already that with CNDO/ $2,{ }^{1}$ INDO, ${ }^{2}$ NDDO, ${ }^{3}$ MINDO/ $2,{ }^{4.5}$ STO$3 \mathrm{G},{ }^{6}$ STO-4-31G, ${ }^{7}$ and some other methods the geometry can be obtained practically accurate in several cases. Second, one must be able to predict the energy associated with different paths of a reaction with a qualitative accuracy, at any rate. Actually it is difficult to estimate the errors involved in quantitative prediction of potential surface, even when rigorous $a b$ initio quantum chemical methods are used. Then, a qualitative prediction of the reaction path is usually the most reliable result of a detailed calculation for a complicated molecular system. Further, as far as the computation of geometry for a number of points of the potential surface is time consuming, a fast-convergence optimization procedure should be elaborated. Finally, an interpretation of the calculated data and the

[^0]data processing with standard procedures allow one to select the criteria for the reaction mechanism.

The quantum mechanical calculation is possible only for an isolated molecular system, for which the mechanism of the energy exchange with the environment cannot be included into the consideration. This approximation holds for intramolecular rearrangements. Those rearrangements were an objective of the detailed quantum chemical calculation recently made. ${ }^{8-11}$

In this work we have investigated the reaction paths for an intramolecular rearrangement in cyclopentadiene and cyclopentadienylsilane.

## Quantum-Chemical Technique Selected

A quantum-chemical method to calculate the reaction coordinate should satisfy at least the following conditions: (A) the calculated ground state geometry should be close to the experimental one; (B) a pattern of the energies associated with different reaction channels should agree qualitatively with the chemical properties; (C) the computation time should be short enough to permit a large, interesting from the point of view of experimental chemistry, molecule to be calculated on the available computer, with the optimization of the geometry variables being included.

Modern $a b$ initio methods usually violate condition C. All three conditions could be satisfied by the semi-
(8) XXIII International Congress of Pure and Applied Chemistry; special lectures presented at Boston (July 26-30, 1971) by M. J. S. Dewar, M. Simonetta, and D. T. Clark.
(9) L. Radom, J. A. Pople, V. Buss, and P. v. R. Schleyer, J. Amer. Chem. Soc., 94, 311 (1972).
(10) D. H. Liskow, C. F. Bender, and H. F. Schaefer, J. Chem. Phys., 57, 4509 (1972).
(11) G. A. Shchembelov and Yu. A. Ustynyuk, Theor. Chim. Acta, 24 389 (1972).
empirical methods EHT, ${ }^{12}$ CNDO/2, ${ }^{1,13}$ INDO, ${ }^{14}$ NDDO, ${ }^{15}$ and MINDO. ${ }^{16,17}$
EHT Technique. The off-diagonal matrix elements are calculated through the formula

$$
\begin{equation*}
H_{i j}=(\mathrm{f}(S) S)_{i j} g\left(H_{i}, H_{j}\right) \tag{1}
\end{equation*}
$$

which is an extension of the Wolfsberg-Helmholtz equation. ${ }^{18}$
Particular versions of eq 1 can hardly satisfy condition A when an EHT version without iteration is employed. We have considered 11 different $f(S)$ functions, e.g., $\mathrm{f}(S)=k, \mathrm{f}(S)=2-|S|$ (cf. the paper ${ }^{19}$ by Cusacks), and four $g\left(H_{i}, H_{j}\right)$ functions, and found no version which would at least qualitatively result in a correct geometry for simple molecules. If condition A is satisfied by applying an iterative variation of $H_{i}$ and $H_{j}$, then the respective EHT technique will gain no advantage of semiempirical MO-SCF methods with respect to condition C.
Semiempirical MO-SCF Techniques. The methods CNDO/2, INDO, and NDDO lead, to all practical purposes and intents, to the same departure (on the average, $4 \%$ ) of the calculated geometrical characteristics from the experimental ones. CNDO is the most widespread technique since it fits with condition $C$ best of all. As calculations of barriers to internal rotation have shown, CNDO/2 may satisfy condition $B$ as well; at any rate, this is so for noncharged molecular systems containing $\mathrm{H}, \mathrm{C}, \mathrm{N}, \mathrm{O}$, and Si atoms.

Recently, Dewar and his coworkers have developed some versions of the MINDO method ${ }^{16,17,20}$ to satisfy not only condition A but also (cf. the numerous calculations in ref 21 ) condition B . Condition C is satisfied by both CNDO/2 and MINDO/2 equally well. However, the MINDO has not yet been extended to embrace elements of the third period of the periodic system. Our main task was to compare driving forces operating in cyclopentadiene and cyclopentadienylsilane rearrangements; therefore, we have limited ourselves to adjusting the calculation to condition B qualitatively and employed the CNDO/2 technique, with the MINDO/2 method being used for check-up purposes in some cases.

## Optimization Procedure

A point on the reaction coordinate requires, in principle, $3 N-6$ geometrical parameters to be calculated for an $N$-atomic molecule. Such a calculation defies even the performance of a modern large computer; therefore convergence inherent in the optimization procedure should be as high as possible. Reportedly, ${ }^{3,22}$
(12) R. H. Hoffimann, J. Chem. Phys., 39, 1397 (1963).
(13) D. P. Santry and G. A. Segal, J. Chem. Phys., 47, 158 (1967).
(14) J. A. Pople, D. L. Beveridge, and P. A. Dodosh, J. Chem. Phys., 47, 2026 (1967).
(15) J. A. Pople, D. P. Santry, and G. A. Segal, J. Chem. Phys., 43, S 239 (1965).
(16) N. C. Baird and M. J. S. Dewar, J. Chem. Phys., 50, 1262 (1969).
(17) M. J. S. Dewar and E. Hasselbach, J. Amer. Chem. Soc., 92, 590 (1970).
(18) M. Wolfsberg and L. Helmholz, J. Chem. Phys., 20, 837 (1952).
(19) L. C. Cusacks, J. Chem. Phys., 43, S 157 (1965).
(20) R. C. Bingham, M. J. S. Dewar, and D. H. Lo, to be submitted for publication; see R. V. Bingham and M. J. S. Dewar, J. Amer. Chem. Soc., 95, 7180 (1973).
(21) N. Bodor and M. J. S. Dewar, J, Amer. Chem. Soc., 94, 9103 (1972), and references therein.
(22) A program based on the simplex algorithm by Brown and Lo; see M. J. S. Dewar and M. C. Kohn, J. Amer. Chem. Soc., 94, 2704 (1972): see also A. Brown, M. J. S. Dewar, and W. Schoeller, ibid., 92, 5516 (1970).
the computation time is $k n^{m}$, where $n$ is the number of parameters to be optimized, $m$ is usually greater than two. We have worked out a procedure characterized by $m=1$ and $k$ usually equal to or lying below six.

Consider $n$ parameters ( $M_{1}{ }^{0}, M_{2}{ }^{0}, M_{3}{ }^{0}, \ldots, M_{n}{ }^{0}$ ) and $n$ steps $\left(d_{1}, d_{2}, \ldots, d_{n}\right)$ of variation of $M$. Calculate total energies $E_{1}, E_{2}$, and $E_{3}$ for $M_{i}{ }^{0}, M_{i}{ }^{0}+d_{i}$, and $M_{i}{ }^{0}-d_{i}$, respectively, with all the other $M$ being fixed. Interpolate the resulting three energies by a parabola and calculate $M_{1}{ }^{(1)}$ corresponding to its minimum. If $\left|M_{i}{ }^{(1)}-M_{i}{ }^{0}\right|>6 d_{i}$, then shift $M_{i}{ }^{(1)}$ by $6 d_{i}$ toward the minimum or else substitute $M_{i}{ }^{(1)}$ for $M_{i}{ }^{0}$. Now, calculate new values of $E_{1}, E_{2}$, and $E_{3}$ for $M_{i}^{(1)}$ if $6 d_{i}>\left|M_{i}{ }^{(1)}-M_{i}{ }^{0}\right|>3 d_{i}$ or for $M_{k}{ }^{0}(k \neq i)$ if $\mid M_{i}{ }^{(1)}-$ $M_{i}{ }^{0}<3 d_{i}$. After all $M_{i}{ }^{(1)}$ are calculated, analyze the differences $\left|M^{(1)}-M^{0}\right|$. If for an $M_{j}{ }^{k}$ the difference is below $\Delta_{j}$ (a preset accuracy), then do not optimize this parameter any longer. Reiterate the procedure applied to $M^{0}$ for all the remaining parameters $M^{(1)}$ and obtain $M^{(2)}$ and so on until all the parameters have been optimized. As a rule, the number of $M^{0}$ is several times that of $M^{(2)}$.
The quantities $\Delta_{i}$ are expressed through $\Delta_{i}=d_{i} / k$ where $k$ is 5,10 , or 20 , depending on the accuracy desired. The magnitudes of $d_{i}$ are equal to $0.01-0.03 \AA$ (distances) and $1-3^{\circ}$ (angles). This optimization technique takes $1 / 8$ to $1 / 15$ the time required by the Brown and Lo program ${ }^{22}$ (see, however, ref 23).

The number of parameters to be optimized may be reduced with the aid of symmetry considerations. The parameters $M^{0}$ for a structure under study may be the geometry parameters obtained for a similar structure with the same quantum chemical method; e.g., CNDO/ 2 usually gives a C-H distance of $1.10-1.12 \AA$, so $M^{(1)}$ (or, in the worst case, $M^{(2)}$ ) is already accurate to $\Delta$. If the starting geometry (the set of parameters $M^{0}$ ) is qualitatively incorrect, the program will automatically start treating another structure by means of rapid scanning with a step of $6 d$ along a few parameters. The calculation accuracy depends on the second derivative of the function $E\left(M_{i}\right)$. In principle, an accuracy of $10^{-9} \AA$ or $10^{-9}$ degree may be achieved quite rapidly. Practically, the CNDO/2 iteration accuracy is $10^{-5} \mathrm{eV}$ with respect to the first occupied level, so the bond lengths are accurate to $10^{-3} \AA$ and the angles to $0.1^{\circ}$. Local extremes ("beats") which sometimes arise in the region of the energy minimum associated with a parameter deteriorate the calculation accuracy and raise the computation time.

## Calculations

The calculations were carried out using a TOREAMOR program written by one of us (G. A. S.) and Gloriozov. The program includes the various versions of EHT, CNDO, INDO, and MINDO methods and a processing of the calculated data with standard procedures. The calculation of one point on the reaction coordinate of cyclopentadiene took approximately 1.5 hr on a BESM6 computer (it is approximately as fast as an IBM360/65).

## Rearrangement Model Selected

The rearrangement in cyclopentadiene is one of a

[^1]

Figure 1. Schematic representation of types of transition states possible in the migration of an organometallic group (M, which may also be hydrogen or a $\mathrm{CH}_{3}$ group) in cyclopentadiene derivatives. GS denotes the ground state.
vast number of degenerate intramolecular rearrangements discovered and studied during the last 10 years. ${ }^{24}$ For organometallic $\sigma$-cyclopentadienyl compounds, in particular, nmr techniques helped to demonstrate that the rearrangement occurs via the 1,2 shift predominantly, ${ }^{24,25}$ that is, it is a [1,5]-sigmatropic rearrangement in terms of the Woodward-Hoffmann notation. ${ }^{26}$ The prototropic rearrangement mechanism in cyclopentadiene and deuterated cyclopentadienes was studied with ir ${ }^{27}$ and $n m r^{28}$ methods in detail.

A priori, three basic mechanisms are possible for the rearrangement of $\sigma$-cyclopentadienyl compounds, ${ }^{11}$ referred to as the transition states $2 \mathrm{C}, 3 \mathrm{C}$, and 5 C in Figure 1.

## Migration via 2C

The coordinate system chosen is shown, and the atoms are numbered, in Figure 2. The $X$ and $Y$ axes lie on the $\mathrm{C}_{11} \mathrm{C}_{7} \mathrm{C}_{8}$ plane.

The $Z$ axis is perpendicular to the plane. The following 26 variables have been optimized: (1) ten bond lengths $R_{i}$ (all the bonds except $\mathrm{C}_{9}-\mathrm{C}_{10}$ ); (2) nine angles $\varphi_{i}$ formed by the atom-atom bonds and the $Y$ axis (all angles of the type, except for the angles associated with $\mathrm{C}_{7}$ and $\mathrm{H}_{6}$ ); (3) five angles $\theta_{i}$ by which the $\mathrm{CH}_{i}$ bonds ( $i \leq 5$ ) deviate away from the plane formed with the carbons nearest to the $\mathrm{H}_{i}$ hydrogens; the angle was assumed positive when the bond deviated toward the atom $\mathrm{H}_{6}$ (the angle $\theta_{6}$ for $\mathrm{H}_{6}$ is the angle between the $Y$ axis and the perpendicular dropped from the point $\mathrm{H}_{6}$ onto the line $\mathrm{C}_{7}-\mathrm{C}_{8}$ ); (4) dihedral angles: $\psi_{1}$ formed by the planes $\mathrm{C}_{7} \mathrm{C}_{8} \mathrm{C}_{10}$ and $\mathrm{C}_{8} \mathrm{C}_{9} \mathrm{C}_{11}, \psi_{2}$ formed by $\mathrm{C}_{9}-$ $\mathrm{C}_{10} \mathrm{C}_{11}$ and $\mathrm{C}_{8} \mathrm{C}_{9} \mathrm{C}_{11}$ (the angles $\psi_{1}$ and $\psi_{2}$ were assumed positive when the planes $\mathrm{C}_{8} \mathrm{C}_{9} \mathrm{C}_{11}$ or $\mathrm{C}_{9} \mathrm{C}_{10} \mathrm{C}_{11}$ deviated toward the atom $\mathrm{H}_{6}$ ).

All these variables were optimized for 16 fixed points lying on the $X$ axis. The bond lengths are accurate to $10^{-3} \AA$ and the angles to $0.1^{\circ}$.

According to their influence upon the energy the geometry parameters may be divided into three types: (i) strongly influential (distances $R(\mathrm{CC})$ and angles $\varphi(\mathrm{CCC})$ between $\mathrm{C}-\mathrm{C}$ bonds), (ii) moderately influential (distance $R(\mathrm{CH})$ and dihedral angles $\psi$ ), (iii) weakly influential (angles $\varphi_{\mathrm{H}}$ and $\theta_{\mathrm{H}}$ ). An increment of $0.02 \AA$ (distances) or 0.02 radian (angles) requires, on the average, the following energies (eV): (i) 0.04 for $R(\mathrm{CC})$

[^2]

Figure 2. The numbering of atoms and the coordinate system chosen for the proton $\mathrm{H}_{6}$ migration in cyclopentadiene.


Figure 3. The pattern of total energy ( $E$ ), dipole moments $(D)$, bond lengths $(R)$, net atomic charges $(q)$, and $\pi$-bond orders ( $\boldsymbol{P}_{i k^{2}}$ ) for the calculated points of the reaction coordinate $(1,2$ shift in cyclopentadiene), with the CNDO/ 2 method.
and $\varphi(\mathrm{CCC})$, (ii) 0.01 for $R(\mathrm{CH}), 0.005$ for $\psi_{1}$ and $\psi_{2}$, (iii) 0.0008 for $\theta_{\mathrm{H}}$, and 0.0005 for $\varphi_{\mathrm{H}}$. The calculation has demonstrated that the geometric variables of the first five hydrogens depend on $X$ weakly. Consequently, the energy change is mainly due to the change in the carbon skeleton and to the shift of $\mathrm{H}_{6}$.

Note that some of the geometrical parameters have sometimes two local minima each, the minima being close in their energies. This is so for $\theta_{\mathrm{H}}$ (at $\theta_{\mathrm{H}}>0$ or $\theta_{\mathrm{H}}<0$ ) or, e.g., for $X=0.60 \AA$ for which there is a minimum at $R(7,11)=1.4554 \AA$ and another one (whose energy is by 0.01 eV higher) at $R(7,11)=1.4674$ $\AA$. These minima interchange their positions at $X=$ $0.75 \AA$; therefore, this point somewhat deviates from the regularities observed for the variation in the parameters associated with the other points on the $X$ axis.


Figure 4. Results (dotted lines) obtained with scanning the parameter, $X$, for fixed points (filled circles) of the reaction coordinate $X$ for $\mathrm{C}_{0} \mathrm{H}_{6}$. Asterisks denote minima of the functions $U_{2}(X)$.


Figure 5. The correlation diagram calculated for the hydrogen 1,2 shift in cyclopentadiene, with respect to the $C_{2 v}$ irreducible representations $\left(a_{1}, a_{2}, b_{1}, b_{2}\right)$; $a_{2}$ is a $\pi$ orbital.

For the ground state ( $X=1.1476 \AA$ ), the minimum at $R(7,11)=1.47 \AA$ turns to lie below the minimum at $R(7,11)=1.467 \AA$; therefore, this state slightly deviates from the $C_{20}$ symmetry. We cannot say whether the presence of two minima is real or is due to an uncertainty inherent in the CNDO/ 2 method.
The reaction coordinate corresponding to the $\mathrm{H}_{6} 1,2$ shift is shown in Figure 3. The state 2C is described by a shallow ( $0.2 \mathrm{kcal} / \mathrm{mol}$ in depth) minimum. The transition state lies at $X=0.174 \AA$. That minimum is, probably, an artifact inherent in the CNDO/2 method. Actually, the method overestimates the energy gain associated with the three-center bonds. ${ }^{29-31}$ Recently, some points on the total energy curve have been cal-
(29) R. Sustmann, J. E. Williams, M. J. S. Dewar, L. C. Allen, and P. v. R. Schleyer, J. Amer, Chem. Soc., 91, 5350 (1969).
(30) H. Fischer and H. Kolmar, Theor. Chim. Acta, 13, 213 (1969).
(31) G. A. Shchembelov, Yu. A. Ustynyuk, and I. P. Gloriozov, Dokl. Akad. Natk SSSR, 214, 362 (1974).
culated for cyclopentadiene, Figure 3 . The results obtained by the authors ${ }^{32}$ fit well with those obtained in this work; however, our preset accuracy was better $\left(10^{-5} \mathrm{eV}\right.$ in the first occupied level energy), so our central minimum is one-half as deep. Another interesting result obtained by the authors ${ }^{32}$ is that the central minimum is at all absent from the potential energy curve calculated through the INDO method. Consequently, the energies resulting from a CNDO/ 2 approach are, apparently, far from quantitative. But these errors of the method do not change our qualitative conclusions (see below) on the physical properties of the computed reaction coordinate. Figure 3 shows the variation in some geometrical parameters, net atomic charges, and $\pi$ bond orders.

The scanning with respect to $X$ has been made for $\mathrm{C}_{5} \mathrm{H}_{6}$, with the geometry variables which correspond to the calculated point on the reaction coordinate being fixed. Figure 4 shows the scanning results (dotted lines) and part of the reaction coordinate (solid line) at $X<0$.

Energy levels as a function of $X$ are shown in Figure 5. On the right, the energy levels are assigned to the $C_{20}$ irreducible representations on the basis of the GS MO coefficients.
The pattern obtained for $\mathrm{C}_{5} \mathrm{H}_{5} \mathrm{SiH}_{3}{ }^{33}$ is qualitatively the same as that for $\mathrm{C}_{5} \mathrm{H}_{6}$. In Table I , the parameters that govern the rearrangement course are collated.

The energy variation along the $Y^{\prime}$ axis is shown in Figure 6. The origin of $Y^{\prime}$ is chosen to lie on the carbon $\mathrm{C}_{10}$; the system of coordinates results from a parallel shift of the system in Figure 2 along the $Y$ axis.

The state 2 C lies at the $Y^{\prime}$ of about $2.30 \AA$. The point $Y^{\prime}=2.81 \AA$ corresponds to an antarafacial transition ( $2 \mathrm{C}_{\mathrm{a}}$ ). This transition cannot, however, re-
(32) J. R. de Dobbelaere, J. W. de Haan, H. M. Buck, and G. I. Visser, Theor. Chim. Acta, 31, 95 (1973).
(33) G. A. Shchembelov and Yu. A. Ustynyuk, J. Organometal. Chem., in press.

Table I. Variation of Some Geometrical Parameters for a Number of Points on the Reaction Coordinate for Rearrangement in Cyclopentadiene and Cyclopentadienylsilane

| $X$ | $\longrightarrow \mathrm{C}_{5} \mathrm{H}_{6}-\longrightarrow-$ |  |  |  | $\bigcirc{ }^{-} \mathrm{C}_{5} \mathrm{H}_{3} \mathrm{SiH}_{3}-\longrightarrow$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\theta^{a}$ | $\theta_{6}$ | $-\theta_{1}$ | $-\theta_{2}$ | $\theta^{a}$ |  | $-\theta_{1}$ | $-\theta_{2}$ |
| 0.00 | 52.6 | 75.8 | 7.0 | 7.1 | 67.6 | 77.5 | 12.5 | 12.5 |
| 0.15 | 57.8 | 74.7 | 12.9 | 4.2 | 69.2 | 75.4 | 16.3 | 10.7 |
| 0.30 | 63.2 | 73.4 | 18.9 | 2.6 | 70.0 | 73.3 | 20.0 | 8.9 |
| 0.45 | 68.2 | 73.2 | 23.4 | 1.6 | 71.4 | 72.5 | 22.7 | 8.9 |
| 0.60 | 70.8 | 72.0 | 29.3 | 1.3 |  |  |  |  |
| 0.75 | 70.7 | 70.7 | 33.9 | 0.9 |  |  |  |  |
| 0.90 | 66.4 | 67.9 | 39.8 | 1.2 |  |  |  |  |
| GS | 59.7 | 59.7 | 51.5 | 0.7 | 70.7 | 70.7 | 31.5 | 2.3 |

${ }^{a}$ The angle formed by the $\mathrm{H}_{6}-\mathrm{C}$ or $\mathrm{Si}-\mathrm{C}$ bond and cyclopentadienyl ring.
sult from continuous variation of geometry parameters with reference to 2 C , since for the atoms $\mathrm{H}_{1}$ and $\mathrm{H}_{2}$ the angles $\theta_{1}$ and $\theta_{2}$ vary from -7 to $-10^{\circ}$ whereas they have opposite signs in the state $2 \mathrm{C}_{\mathrm{a}}$. Accordingly, the points $Y^{\prime}=2.70$ and 2.81 are joined by a dotted line.

Moving from $2 \mathrm{C}_{5}$ (s denotes suprafacial) leftward strongly raises the angle $\psi_{1}$ (its value is shown near the points), with the atom $\mathrm{C}_{10}$ meeting $\mathrm{H}_{6}$; the angle $\theta_{1}$ varies from -7.1 to $-14.1^{\circ} ; \theta_{3}$ from -1.4 to $-9.3^{\circ}$; $\theta_{4}$ from zero to $-7.7^{\circ}\left(Y^{\prime}=1.10\right)$ and then to $-1.0^{\circ}$ ( $Y^{\prime}=0.10$ ) ; $R\left(\mathrm{C}_{7}-\mathrm{H}_{6}\right)$ varies from 1.220 to $1.349 \AA$; $R\left(\mathrm{C}_{9}-\mathrm{H}_{10}\right)$ from 1.392 to $1.422 \AA$; the remaining geometry variables are practically not affected. In going from 2 C to the point $Y^{\prime}=0.10 \AA$, the charge on $H_{6}$ falls from 0.17 down to -0.07 , the dipole moment rises from 2.80 up to 4.50 D . These data suggest that the state 5 C , of the $C_{50}$ symmetry ( $Y^{\prime}=1.206 \AA$ ), is inaccessible if the molecule starts from 2C.

For $\mathrm{C}_{5} \mathrm{H}_{5} \mathrm{SiH}_{3}$, the pattern is qualitatively the same, although the parameters vary in a less pronounced fashion. Moving leftward from $2 \mathrm{C}_{\mathrm{s}}$ up to $Y^{\prime}=0.90$ $\AA$ raises the angle $\psi_{1}$ to $20.9^{\circ}$ and the $\Delta E$ value by 1.22 eV.


Figure 6. $\Delta E$ vs. $Y^{\prime}$ for cyclopentadiene via $\mathrm{CNDO} / 2$. Left-hand curve, the motion starts from GS. Right-hand curve, same from 2 C. The figures near the points are the dihedral angle $\psi_{1}$ values.
the orbital types are affected by the rearrangement. Table II lists the values of $Q_{A B}=\left(\Pi_{i} d_{i}\right)^{1 / 2}$ for some points on the reaction coordinate $X$ for $\mathrm{C}_{5} \mathrm{H}_{6}$.

Table III lists the $d_{i}$ values that lie below 0.99. Let $x y$ denote the set of coefficients $C_{i \mu}$ of the orbital $\psi_{i}=$ $\Sigma_{\mu} C_{i \mu} \chi_{\mu}$, the coefficients which correspond to the $\chi_{\mu}=$ $2 \mathrm{p}_{x}, 2 \mathrm{p}_{y}$ carbon orbitals, and let $\pi$ denote the set of $2 \mathrm{p}_{2}$. The molecular orbitals in Table III will correspond to the following interaction types: no, $8,\left(\mathrm{H}_{6}\right.$ $x y$ ) for GS and 2C; no. 11, ( $\left.\mathrm{H}_{1} \mathrm{H}_{6} / \pi\right)$ for GS, $\left(\mathrm{H}_{i} x y\right)$ for 2C; no. 12, $\left(\mathrm{H}_{i} \mid x y\right)$ for GS, $\left(\mathrm{H}_{6} \mid \pi\right)$ for 2 C ; no. 13, ( $\pi \mid \pi$ ) for GS and 2C; $i \leq 5$ throughout.

## Migration via 5 C

The [1,3]-rearrangement channel, allowed antarafacially, and the [1,1]-rearrangement channel, allowed

Table II. $Q_{A B}$ for Geometry Configurations Corresponding to Various Points on the Reaction Coordinate $X$ for $\mathrm{C}_{6} \mathrm{H}_{5}$

| $X$ | 1.05 | 0.90 | 0.75 | 0.60 | 0.45 | 0.30 | 0.15 | 0.00 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.14 (GS) | 0.996 | 0.983 | 0.965 | 0.944 | 0.915 | 0.877 | 0.821 | 0.749 |
| 1.05 |  | 0.996 | 0.984 | 0.968 | 0.942 | 0.908 | 0.853 | 0.781 |
| 0.90 |  |  | 0.996 | 0.987 | 0.967 | 0.938 | 0.887 | 0.817 |
| 0.75 |  |  |  | 0.996 | 0.984 | 0.961 | 0.916 | 0.849 |
| 0.60 |  |  |  |  | 0.995 | 0.979 | 0.943 | 0.884 |
| 0.45 |  |  |  |  |  | 0.994 | 0.970 | 0.921 |
| 0.30 |  |  |  |  |  |  | 0.990 | 0.957 |
| 0.15 |  |  |  |  |  |  |  | 0.987 |

## Reaction Coordinate Points Processed with the Corresponding Orbitals Method

In terms of the corresponding orbitals method, ${ }^{34,35}$ a criterion for the "permissibility" of the transition between the geometrical configurations A and B of a molecular system is the matrix $D_{i k}=\left(\psi_{i}{ }^{\mathrm{A}} \mid \psi_{k}^{\mathrm{B}}\right)$ where $\psi_{i}{ }^{A}$ is the $i$ th occupied orbital in configuration A . On diagonalizing $\mathrm{D}^{+} \mathrm{D}$ we obtain a set of roots, $d_{i}$, and deviations of the roots from unity will indicate which of

[^3]suprafacially, are both impossible owing to spatial considerations.

Figure 6 demonstrates that the state 5 C of the $C_{5}$ v symmetry ( $Y^{\prime}=1.206 \AA$ ) is inaccessible if the molecule starts from GS $\left(Y^{\prime}=-0.67 \AA\right)$, while for the 3 C state there is no minimum. In their motion from GS, the atoms $\mathrm{C}_{10}$ and $\mathrm{H}_{6}$ move in the opposite directions; the hydrogen $\mathrm{H}_{4}$ follows the direction of $\mathrm{H}_{6}$. By collating this to the $\mathrm{H}_{6}$ motion starting from GS, we obtain that the [ 1,1 ] shift along the route: $\mathrm{GS} \rightarrow 3 \mathrm{C} \rightarrow 5 \mathrm{C} \rightarrow$ $2 \mathrm{C}_{8} \rightarrow 2 \mathrm{C}_{\mathrm{a}}$ and further (at the back side of the ring) $2 \mathrm{C}_{8} \rightarrow 5 \mathrm{C} \rightarrow 3 \mathrm{C} \rightarrow \mathrm{GS}$ is impossible, since the curves in

Table III. Roots of Matrix $D+D$ for $\mathrm{C}_{5} \mathrm{H}_{6}$



Figure 7. Correlation diagram for energy levels for the route GS $\rightarrow$ $3 \mathrm{C} \rightarrow 5 \mathrm{C} \rightarrow 2 \mathrm{C}_{\mathrm{s}} \rightarrow 2 \mathrm{C}_{\mathrm{a}}$ : ( + ) nondegenerate levels for 5 C , ( $O$ ) and (一) degenerate levels for 5 C when the motion starts from GS, (. - $)$ and $(-,-)$ degenerate levels for 5 C when the motion starts from $2 \mathrm{C},(\cdots)$ correlation with the state 5 C (the $C_{s v}$ symmetry).

Figure 6 do not intercept because they lie on different sections of the potential surface (the different angles $\psi_{1}$ ). The same conclusion follows from Figure 7 .
The corresponding orbitals approach shows that in the transition under discussion the most affected is orbital no. 11. It is in the form of $\left(\mathrm{H}_{1} \mathrm{H}_{6} \mid \pi\right)$ for GS, $\left(\mathrm{H}_{i} \mid x y\right)$ for 2 C , while $\mathrm{H}_{6}$ takes no part in it in the case of

3 C and 5 C . For $\mathrm{C}_{5} \mathrm{H}_{5} \mathrm{SiH}_{3}$, the last occupied orbital is affected; it has the form of $\left(\mathrm{p}_{z}(\mathrm{Si}) \mid \pi\right)$ for GS while for 5 C it coincides with the last occupied $\pi$ orbital and forms a degenerate $\pi$ orbital. The values of $Q(\mathrm{GS}, 5 \mathrm{C})$ are 0.565 and 0.659 for $\mathrm{C}_{\dot{3}} \mathrm{H}_{6}$ and $\mathrm{C}_{5} \mathrm{H}_{5} \mathrm{SiH}_{3}$, respectively.

## Discussion

The calculation demonstrates that the state 5 C is inaccessible, since the atoms M and C of the M-C bond ( M is $\mathrm{H}, \mathrm{SiH}_{3}$ ) move along the opposite directions when they start from GS. Thus, there is no reaction coordinate that could have corresponded to the $[1,3]$ or [1,1] shifts of the group M. In the final analysis, the motion from GS to 5C leads to dissociation of the molecule resulting in the ions $\mathrm{M}^{+}$and $\left(\mathrm{C}_{5} \mathrm{H}_{5}\right)^{-}$.

The state 5C is not accessible either when the motion starts from 2C; this motion leads, practically, to the dissociation resulting in the radicals M - and $\left(\mathrm{C}_{\mathrm{j}} \mathrm{H}_{5}\right)$.

For the symmetry-forbidden antarafacial [1,5] shift, for which the group M and the four carbons lie on the same plane (the state $2 \mathrm{C}_{3}$ ), there is no reaction coordinate either.

For the symmetry-allowed suprafacial [1,5] shift, we have obtained a potential curve (Figures 3 and 4) close to the reaction coordinate. The energies of activation obtained by CNDO/ 2 for the degenerate rearrangements in $\mathrm{C}_{5} \mathrm{H}_{6}$ and $\mathrm{C}_{5} \mathrm{H}_{5} \mathrm{SiH}_{3}$ are 10.2 and $6 \mathrm{kcal} / \mathrm{mol}$, respectively. They are approximately twice lower than the experimental values ( 24 and $13 \mathrm{kcal} / \mathrm{mol}$, respectively). ${ }^{25,36}$ The energy of activation obtained through MINDO/2 (ref 17 ) is $18.4 \mathrm{kcal} / \mathrm{mol}$. Although the results provided by the two methods differ in terms of the potential surface energy pattern, the optimized geometries coincide quantitatively. ${ }^{11,31,32}$ The calculated geometry fits to the experimental one ${ }^{37-39}$ not only qualitatively but also quantitatively well. The highest

[^4]departure ( $2.5 \%$ ) is observed for $R(7,8)$. The calculation shows that when GS is replaced with 2 C the angles $\theta_{1}, \theta_{2}$, and $\theta_{6}$ are affected to the greatest extent, the carbon skeleton parameters are less affected, whereas the parameters of the atoms $\left(\mathrm{H}_{3}, \mathrm{H}_{4}\right.$, and $\left.\mathrm{H}_{5}\right)$ farthest from the migrating $\mathrm{H}_{6}$ remain almost unaffected. The $R(7,8)$ bond length associated with the carbons between which the hydrogen $\mathrm{H}_{6}$ migrates does not undergo any significant change either.

The following three regions in Figure 3 are worth special discussion: (A) minima, where energy is a quadratic function of $X, 1.35>X>0.90$; (B) descents, where the energy change is practically linear, $0.90>$ $X>0.35$; (C) passes, where energy is almost constant, $0.30>X>-0.30$.

Region A (Ground State). The most sensitive variables are the angles $\theta_{1}, \theta_{6}$, and $\psi_{1}$ (they change from $-51.5,59.7$, and $0.7^{\circ}$ to $-33,71$, and $5.4^{\circ}$ respectively) and the orbitals no. 8 and 11 associated with the interactions $\left(\mathrm{H}_{6} \mid x y\right)$ and $\left(\mathrm{H}_{1} \mathrm{H}_{6} \mid \pi\right)$.

Region B (Intermediate). The interaction ( $\left.\mathrm{H}_{1} \mathrm{H}_{6}{ }^{1} \pi\right)$ (orbital no. 12) starts rising. Consequently, the angle $\varphi_{8}$ (or the valence angle $\varphi(11,7,8)$ ) increases markedly (by $3^{\circ}$ ). Positive charge on $\mathrm{H}_{6}$ augments strongly (from 0.063 to 0.137 ) while negative charges on $\mathrm{C}_{9}$ and $\mathrm{C}_{11}$ decrease. The electric moment and the $\mathrm{H}_{6}-\mathrm{C}_{7}$ distance increase noticeably.

Region C (Transition State). The ring $\pi$ orbital (no. 13) changes strongly. Therefore, all the $\mathrm{C}-\mathrm{C}$ bond lengths (except for $R(7,8)$ ) and, in parallel, the $\pi$ bond orders are affected. The dihedral angle $\psi_{1}$ falls from 3.7 down to $0^{\circ}$; the angle $\psi_{2}$ increases from 0.2 up to $1.8^{\circ}$. The $\mathrm{C}_{7}-\mathrm{H}_{6}$ distance becomes $0.1 \AA$ longer. The electric moment and the charges on $\mathrm{H}_{6}, \mathrm{C}_{9}$, and $\mathrm{C}_{11}$ rise. The quantities $Q_{A B}$ are affected to the greatest extent in the region under discussion; therefore this region may be the least accessible.

In going from GS to 2 C , the charge on $\mathrm{H}_{6}$ jumps from 0.028 up to 0.165 . The molecular dipole moment rises from 0.84 D in GS to 2.80 D in 2 C . The experimental value ${ }^{40}$ is 0.53 D . For $\mathrm{C}_{5} \mathrm{H}_{5} \mathrm{SiH}_{3}$, the reaction coordinate behavior is qualitatively the same (the
(40) N. B. Hanney and C. P. Smith, J. Amer. Chem. Soc., 68, 244 (1946).
depth of the shallow minimum at the pass is in this case within the calculation accuracy). A difference relates to the $d_{i}$ values, of which the following undergo the most significant alteration: $d_{16}=0.823, d_{15}=0.886$, $d_{13}=0.915$, and $d_{12}=0.958$. They correspond to the following interactions: $(\pi \mid \pi) ;\left(\mathrm{p}_{2}(\mathrm{Si}) \mid \pi\left(\mathrm{C}_{5} \mathrm{H}_{5}\right)\right) ; \quad\left(\mathrm{p}_{x^{-}}\right.$ $\left.\mathrm{p}_{\nu}(\mathrm{Si}) \mid \mathrm{p}_{x} \mathrm{p}_{\nu}\left(\mathrm{C}_{5} \mathrm{H}_{5}\right)\right)$; and $\left(\mathrm{H}_{1} \mathrm{H}_{2} \mid \mathrm{p}_{x} \mathrm{p}_{y}\left(\mathrm{C}_{5} \mathrm{H}_{5}\right)\right)$, ( $\mathrm{p}_{x} \mathrm{p}_{y} \mathrm{p}_{z^{-}}$ (Si) $\left.\mathrm{p}_{x} \mathrm{p}_{y}\left(\mathrm{C}_{5} \mathrm{H}_{5}\right)\right) . \quad Q(\mathrm{GS}, 2 \mathrm{C})$ is 0.793 . This difference is due to the fact that silicon has $p$ orbitals and may relate to the variation in the cyclopentadienyl skeleton (see tables in ref 11 and 33), the angles $\theta, \theta_{1}$, and $\theta_{2}$ (Table I).

Figure 4 allows one to make the following conclusions. (1) The reaction coordinate is an envelope of a set of functions $U_{1}, U_{2}, \ldots, U_{n}$ of one variable. The set of minima of the functions $U_{i}$ has its upper and lower limits. (2) The curves $U_{i}$ are unsymmetrical, except for $X=0$. The unsymmetry prompts that there is a trend for the transition into 2 C at region C ; the trend for GS holds at region $A$. The curves are the most sensitive in the transient $B$ region.

The data in Figure 4 can be interpreted only after additional calculations have been done. ${ }^{41}$

## Conclusion

There is no reaction coordinate for the $[1,3]_{\mathrm{a}}-,[1,1]_{s}$, and $[1,5]_{a}$-sigmatropic-rearrangement mechanisms forbidden by symmetry and spatial considerations in cyclopentadiene or cyclopentadienylsilane. The configuration of the $C_{5 x}$ local-site symmetry is inaccessible, and the motion thereto leads, practically, to the dissociation resulting in ions or radicals.

A potential curve close to the reaction coordinate has been calculated for the symmetry-allowed [1,5]s-rearrangement channel. Whether or not the rearrangement will be facile depends on (i) the angle formed by the $\mathrm{M}-\mathrm{C}$ bond ( M is $\mathrm{H}, \mathrm{SiH}_{3}$ ) and the cyclopentadienyl ring plane, (ii) the angles by which the $\mathrm{C}-\mathrm{H}$ bonds nearest to the $\mathrm{M}-\mathrm{C}$ bond deviate away from the ring plane, and (iii) leveling of $\mathrm{C}-\mathrm{C}$ bonds in the ring.

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[^5]
[^0]:    (1) J. A. Pople and G. A. Segal, J. Chem. Phys., 44, 3289 (1967).
    (2) M. S. Gordon and J. A. Pople, J. Chem. Phys., 49, 4643 (1968).
    (3) R. B. Davidson, W. L. Jorgensen, and L. C. Allen, J. Amer. Chem. Soc., 92, 749 (1970).
    (4) N. Bodor, M. J. S. Dewar, and D. H. Lo, J. Amer. Chem. Soc., 94, 5303 (1972).
    (5) M. J. S. Dewar and D. H. Lo, J. Amer. Chem. Soc., 94, 5296 (1972).
    (6) M. D. Newton, W. A. Lathan, W. J. Hehre, and J. A. Pople, J. Chem. Phys., 52, 4064 (1970).
    (7) R. Ditchfield, W. J. Hehre, and J. A. Pople, J. Chem. Phys., 54, 724 (1971).

[^1]:    (23) A. Brown, M. J. S. Dewar, H. Metiu, and J. Wasson, to be submitted for publication: see R. C. Bingham and M. J. S. Dewar, J. Amer. Chem. Soc., 95, 7180 (1973).

[^2]:    (24) F. A. Cotton, Accounts Chem. Res., 1, 257 (1968).
    (25) A. V. Kisin, V. A. Korenevsky, N. M. Sergeyev, and Yu. A. Ustynyuk, J. Organometal. Chem., 34, 93 (1972).
    (26) R. B. Woodward and R. Hoffmann, "The Conservation of Orbital Symmetry," Verlag Chemie, Weinheim/Bergstr., Germany, 1970.
    (27) V. A. Mironov, E. V. Sobolev, and A. N. Elizarova, Tetrahedron, 19, 1939 (1963).
    (28) W. R. Roth, Tetrahedron Lett., 1009 (1964).

[^3]:    (34) H. F. King, R. E. Stanton, H. Kim, R. E. Wyatt, and R. G. Parr, J. Chem. Phjs., 47, 1936 (1967).
    (35) C. Trindle, J. Amer. Chem. Soc., 92, 3251 (1970).

[^4]:    (36) S. McLean and P. Haynes, Tetrahedron, 21, 2329 (1963).
    (37) G. Liebling and R. E. Marsh, Acta Crj siallogr.. 19, 202 (1965),
    (38) L. H. Scharpen and V. W. Laurie, J. Chem. Phis., 43, 2765 (1965).
    (39) J. E. Bentham, D, W, H. Rankin, J. Organometal. Chem., 30, C54 (1971).

[^5]:    (41) G. A. Shchembelov, results in preparation for publication.

